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Radiative exchange of heat between nanostructures

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Abstract. Surfaces in close proximity exchange heat through evanescent photon tunnelling modes as well as by freely propagating modes. These additional near field contributions to radiation scale with separation, d, between surfaces as d^{-2} and are dominant at spacings $d \ll \lambda_T$, a typical wavelength at temperature T. We calculate simple expressions for the photon tunnelling and find that there are drastic effects in many nanostructured systems, for example in the scanning tunnelling microscope. The results are linked to quantum information theory which dictates that the maximum heat tunnelling current in any one channel is determined by the temperature alone. Consequences for the scanning tunnelling microscope are discussed, where a hot tip may cause intense local heating of a surface without actually being in physical contact, hence desorbing molecular species, or even modifying the surface itself: a possible extension of the STM's capacity for surface modification. A further extension of this concept is proposed in the form of a 'heat stamp' capable of delivering a high definition pattern of heat to a second surface.

1. Introduction

Every physics undergraduate knows Stefan's law [1] which states that a hot black surface will radiate according to the law,

$$\dot{Q}_{BB} = \frac{\pi^2 k_B^4}{60\hbar^3 c_0^2} T^4 \tag{1}$$

where k_B is Boltzmann's constant, $\hbar$ Planck's constant divided by 2π and c_0 the velocity of light in free space. Two black bodies with parallel surfaces (figure 1) will exchange heat according to this law independent of the distance, d, between them. The vacuum can be said to have a thermal conductivity.

In this paper we point out that heat transport across the vacuum is not independent of d when,

$$d < \lambda_T = \frac{2\pi c_0 \hbar}{k_B T}. \tag{2}$$

λ_T is the wavelength characteristic of temperature T. We shall show that outside the surface of a dissipative material there are electromagnetic modes that decay exponentially into the vacuum. Provided that the surfaces are close enough, heat can also be transported by photons tunnelling through evanescent modes. At low temperatures of a few K it is possible for this form of transport to be dominant even at spacings of a few mm, see table 1.

We imagine that photon tunnelling is important in situations where surfaces are in close proximity *in vacuo*. For example if two otherwise flat surfaces are pressed together in the presence of microscopic insulating granules, then heat conduction can be expected to be dominated by tunnelling. Alternatively we might consider nanoscopic structures so common

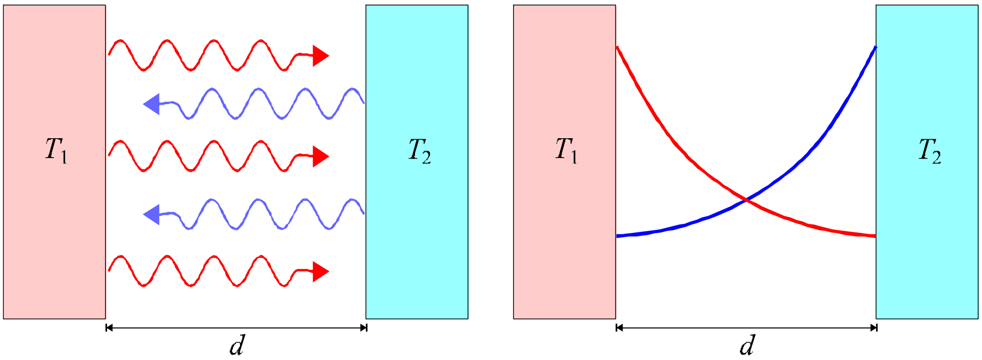

Figure 1. There are two modes for exchange of heat between two surfaces separated by vacuum: conventional radiative transfer, or photon tunnelling via evanescent states. The latter dominate at short distances. See table 1.

Table 1. Critical distance for evanescent waves to dominate. At distances of a few nanometres, radiative heat flow is almost entirely due to evanescent modes.

T (K)	λ_T (μm)
1	2289.8
4.2	545.2
100	22.9
273	8.4
1000	2.3

in experimental electronic devices, or the scanning tunnelling microscope [2], where all the relevant dimensions are no more than a few nm. In a related device, the scanning near field optical microscope [3, 4], light is carried from one medium to the next by evanescent modes. For objects that are very close together the black body radiation formula is completely irrelevant to heat transport, and photon tunnelling will be the dominant mode. Interaction between the STM and electromagnetic radiation has been demonstrated at optical frequencies [5, 6].

Photon tunnelling was considered some time ago by Polder and Van Hove [7] who calculated the tunnelling between two flat surfaces. However, although their expression for heat conduction is equivalent to mine, their approach was very different. The present derivation offers an alternative insight and is a little more compact. Since the pioneering work of Polder and Van Hove experiments on nanostructures have come to prominence greatly increasing the relevance of photon tunnelling which dominates heat transfer between these structures. It is now possible to measure extremely small amounts of heat transfer into small volumes [8]. In later sections of the paper we consider different geometries which may be relevant to nanostructures such as a sphere outside a surface. We also show links between photon tunnelling and the recently explored quantum friction [9, 10].

A striking new result we find is the limiting heat flux that a single channel can sustain. Analogous to the maximum radiative power that a black body provides, the conditions for limiting heat flux for tunnelling modes are quite different. The existence of a maximum heat flux was predicted some time ago [11] on very general grounds of quantum information theory and the limiting flux we calculate here is consistent with the earlier work.

2. Tunnelling between two transparent dielectrics

The concept of photon tunnelling can be neatly illustrated by considering a transparent dielectric such as glass: see figure 2. Within the dielectric black body radiation has a higher density than in vacuum as can be seen from formula (1): if the velocity of light is reduced, the density of radiation increases. The extra radiation is contained in waves that have large wave vectors parallel to the lower surface. Since parallel wave vector is conserved across a flat surface these modes cannot escape into vacuum and experience total internal reflection. Thus the surface carefully rejects just the right amount of radiation to ensure that the density of black body radiation emerging into vacuum does not exceed that allowed by formula (1).

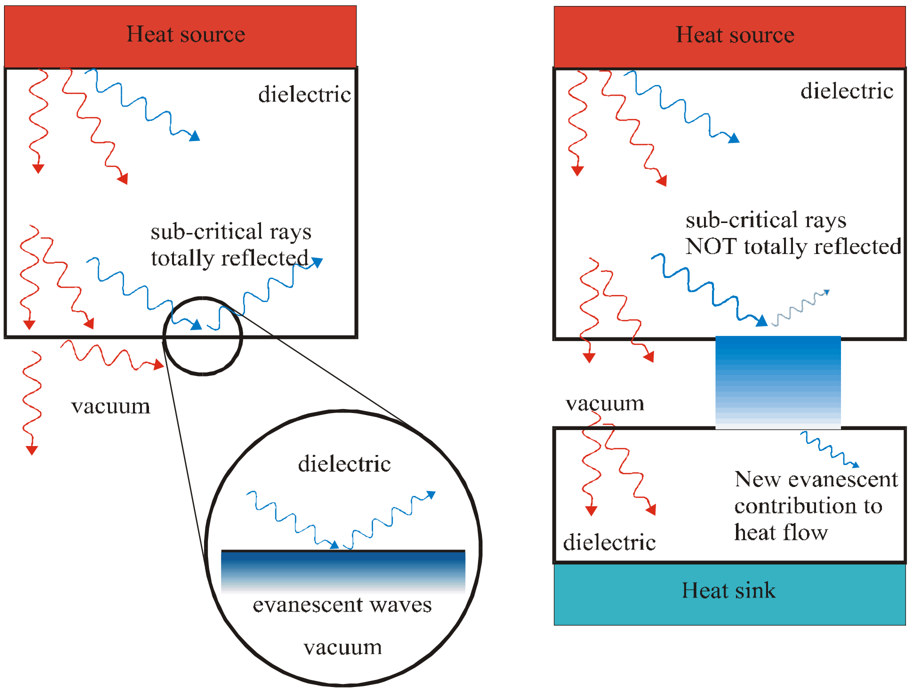

Figure 2. Evanescent waves play no role in heat loss from a hot dielectric surface to vacuum, left hand figure, but evanescent waves can carry heat from a hot to a cold dielectric surface, right hand figure.

It is well known that a second dielectric, if close enough to the first one, will allow the total internal reflection condition to be relaxed as some of the photons tunnel across into the second medium. In the limit that the two dielectrics are in contact all the modes tunnel across without impediment.

The introduction of tunnelling modes associated with large wave vectors parallel to surfaces makes an important point: each wave vector corresponds to a channel, or mode, through which heat can flow. Very good conductors of heat are characterized by having many channels. For example heat flow through a solid may be via the phonons as in sapphire. In general phonons have much smaller wave vectors at a given frequency than does light and therefore there are many more modes accessible at a given temperature for transport of heat.

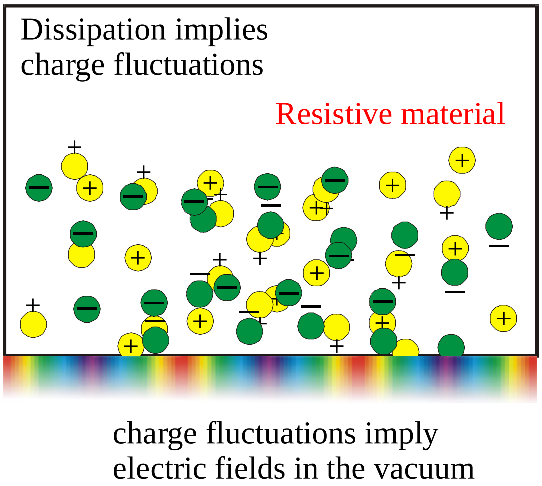

Figure 3. Resistive media also have evanescent waves outside their surfaces. In fact they are a much more potent source of evanescent waves because they support very short wavelength states not found near dielectrics. These fluctuations are the nanoscopic equivalent of Johnson noise.

For light the wave vector normal to a surface is given by

$$K_z = +\sqrt{\omega^2 \varepsilon c_0^{-2} - k^2} \tag{3}$$

where ω is the frequency of the light, ε the dielectric function of the medium and k the wave vector parallel to the surface. Obviously K_z ceases to be real when

$$k > \omega c_0^{-1} \sqrt{\varepsilon} \tag{4}$$

and large wave vector modes are thus not available for radiative transfer of heat. If we introduce tunnelling the situation is quite different. If only we can activate the large wave vector modes, and require that the distance they have to transport the heat is very small, it does not matter that the waves decay exponentially.

If we seek materials that have a high density of large wave vector modes immediately outside their surfaces, transparent dielectrics are not good candidates. Maybe if the dielectric function is large a few more modes can be squeezed into the picture but generally speaking we must look to other materials for really large effects.

The key materials are resistive conductors. Within a resistor fluctuations in electron density create a high density of electromagnetic fields. One instance of these fields is the Johnson noise that is measured across a resistor at finite temperatures. Since electrons may have very large wave vectors it is often the case that the density fluctuations extend to short wavelengths and hence so do the electromagnetic fluctuations: see figure 3.

By tuning the resistivity of the material we can optimize the number of short wavelength modes and hence the potential for heat transport by tunnelling. The energy density at distance d for wave vector k and frequency ω is proportional to

$$\operatorname{Im} R_p(k, \omega) \exp(-kd) \tag{5}$$

where $R_p(k, \omega)$ is the reflection coefficient of the surface at wave vector k and frequency ω, and

$$\operatorname{Im} R_p(k, \omega) \approx \frac{2\sigma/\omega\varepsilon_0}{4 + (\sigma/\omega\varepsilon_0)^2} \qquad k \gg \frac{\omega}{c_0}. \tag{6}$$

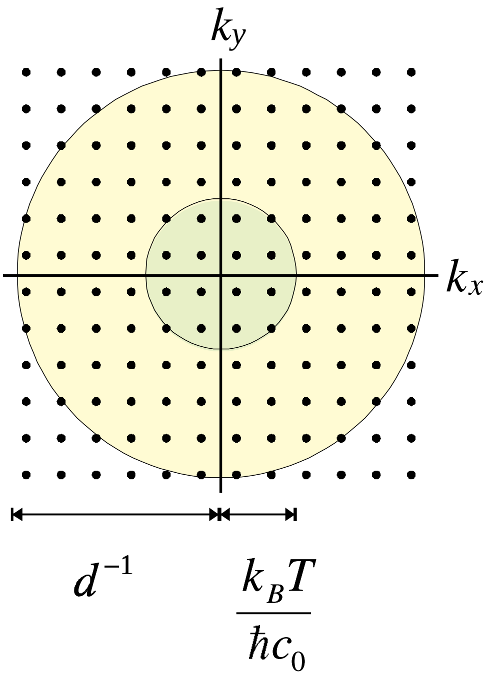

Figure 4. The importance of short wavelength fluctuations: at short distances, evanescent states dominate in phase space: propagating photon modes carry heat flux within the inner circle, evanescent modes within the outer circle.

It is generally assumed that the conductivity, σ, is independent of (k, ω) over a wide range of values.

The energy density is a maximum when

$$\sigma_{\max} = 2\omega\varepsilon_0 \approx \frac{2k_B T \varepsilon_0}{\hbar} = 2.3T\ (\mathrm{m}\Omega)^{-1} \tag{7}$$

where we have substituted a frequency typical of temperature T. At room temperature the optimum electrical conductivity is 690 $(\mathrm{m}\Omega)^{-1}$.

Figure 4 makes the point that the number of channels available to conduct heat may be very much larger for the evanescent modes than for the propagating modes provided that the objects concerned are separated by a short distance.

3. Calculating the tunnelling between two surfaces

Consider two flat surfaces, parallel to each other, separated by distance d. They have reflection coefficients $R_1(k, \omega)$, $R_2(k, \omega)$. Suppose that surface 1 'emits' a p-polarized electromagnetic wave whose electric field is given by

$$\boldsymbol{E}_p = E_{0p}\hat{\boldsymbol{K}}_p^+ \exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}_\parallel - \alpha z) \tag{8}$$

where

$$\alpha = +\sqrt{k^2 - \omega^2 c_0^{-2}} \tag{9}$$

and

$$\hat{\boldsymbol{K}}_p^\pm = \frac{\mathrm{i}c_0\alpha}{\omega k}[k_x, k_y, \pm\mathrm{i}k^2\alpha^{-1}] \tag{10}$$

is the polarization vector normalized to unity,

$$\hat{\boldsymbol{K}}_p^+ \cdot \hat{\boldsymbol{K}}_p^+ = 1. \tag{11}$$

Note that we already assume that we are in the regime of decaying waves, i.e.,

$$k^2 > \omega^2 c_0^2. \tag{12}$$

Reflection from the second surface, and further contributions from multiple reflections between the two surfaces modify the wavefield,

$$\begin{aligned} \boldsymbol{E}'_p = E_{0p}[\hat{\boldsymbol{K}}_p^+ \exp(\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{r}_\parallel - \alpha z) + R_{2p}(k, \omega)\hat{\boldsymbol{K}}_p^- \exp(\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{r}_\parallel + \alpha z - 2\alpha d)] \\ \times [1 - R_{1p}(k, \omega) R_{2p}(k, \omega)]^{-1}. \end{aligned} \tag{13}$$

We assume for simplicity that polarization is conserved and leave generalization to optically active media as an exercise.

Calculating the Poynting vector for this field gives the flow of energy,

$$\dot{q}_p(k, \omega) = \frac{2|E_{0p}|^2}{\omega \mu_0} \alpha\, \mathrm{e}^{-2\alpha d} \frac{\operatorname{Im} R_{2p}(k, \omega)}{|1 - R_{1p}(k, \omega) R_{2p}(k, \omega)\, \mathrm{e}^{-2\alpha d}|^2}. \tag{14}$$

The question arises of what is the value of $|E_{0p}|^2$? This we calculate by using the electromagnetic Green function to calculate the density of states outside the first surface [9], then populating each state with energy

$$\hbar|\omega| \left[\frac{1}{2} + \frac{1}{\exp(\beta \hbar |\omega|) - 1}\right] \tag{15}$$

to give

$$|E_{0p}|^2 = \hbar|\omega| \left[\frac{1}{2} + \frac{1}{\exp(\beta \hbar |\omega|) - 1}\right] \frac{\omega \operatorname{Im} R_{1p}(k, \omega)}{\varepsilon_0 c_0^2 \alpha} \frac{2}{\pi}. \tag{16}$$

Hence,

$$\dot{q}_p(k, \omega) = \frac{8\hbar|\omega|}{\pi} \left[\frac{1}{2} + \frac{1}{\exp(\beta \hbar |\omega|) - 1}\right] \mathrm{e}^{-2\alpha d} \frac{\operatorname{Im} R_{1p}(k, \omega) \operatorname{Im} R_{2p}(k, \omega)}{|1 - R_{1p}(k, \omega) R_{2p}(k, \omega)\, \mathrm{e}^{-2\alpha d}|^2}. \tag{17}$$

The expression for tunnelling through the s-polarized channel is exactly the same except that R_s (the s-polarized reflection coefficient) is substituted for R_p. However note that for evanescent states R_s and R_p are very different and for large k the p-channel is predominantly electrostatic and is dominant unless we are dealing with a magnetic material where the s-channel is also active.

The complete heat transfer from surface 1 to 2 via evanescent states is

$$\dot{Q}_{EV}(T, d) = \sum_{\boldsymbol{k}} \int_0^\infty [\dot{q}_p(k, \omega) + \dot{q}_s(k, \omega)]\, \mathrm{d}\omega. \tag{18}$$

This completes the formal derivation.

Our formula for heat flow, (17), is reminiscent of the formula for quantum friction between two moving surfaces [9]. In both cases the coupling between the surfaces is given by the overlap of the density of states, as modified by multiple scattering between the two surfaces expressed in the final fraction in (17).

4. Maximum heat flow in a channel and quantum information theory

In the case of heat transfer through free photons, the transfer is maximal when both bodies are perfectly black and have zero reflection coefficient,

$$R_i^2 + R_r^2 = 0 \qquad k < \omega c_0^{-1}. \tag{19}$$

What is the photon tunnelling equivalent of a black body? For $k > \omega c_0^{-1}$ there are no constraints on the reflection coefficient $R(k, \omega)$ other than that Im $R(k, \omega)$ is positive. Therefore, assuming identical surfaces, we are free to maximize

$$X = \frac{\mathrm{e}^{-2\alpha d}(\mathrm{Im}\, R)^2}{|1 - R^2\,\mathrm{e}^{-2\alpha d}|^2}. \tag{20}$$

The result is very simple: the expression is a maximum when

$$R_i^2 + R_r^2 = \mathrm{e}^{2\alpha d} \qquad k > \omega c_0^{-1} \tag{21}$$

so that

$$X = \tfrac{1}{4}. \tag{22}$$

Substituting gives

$$[\dot{Q}_{EV}]_{\max} = \sum_{\boldsymbol{k}} \frac{\pi k_B^2}{3\hbar} T^2. \tag{23}$$

In the electrostatic limit, $k > \omega c_0^{-1}$,

$$R_p(\omega) \approx \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 1}.$$

we can specify a dielectric function whose surface reflectivity satisfies the maximum condition:

$$\frac{2\varepsilon_r}{\varepsilon_i^2 + \varepsilon_r^2 + 1} = -\tanh(\alpha d) \tag{24}$$

or explicitly,

$$\varepsilon_r = -\frac{1}{\tanh(\alpha d)} \pm \sqrt{\left(\frac{1}{\tanh(\alpha d)}\right)^2 - (\varepsilon_i^2 + 1)}. \tag{25}$$

At $d = 0$ the condition is even simpler,

$$\varepsilon_{\max} = (\varepsilon_r = 0) + \mathrm{i}\varepsilon_i. \tag{26}$$

Note that this choice of ε definitely does not result in a surface that absorbs all the propagating photons, even though it optimizes flow in the photon tunnelling channels. The optimum choice is realized at low frequencies for materials having finite conductivity, σ, where

$$\varepsilon_{metal} = 1 + \mathrm{i}\frac{\sigma}{\omega\varepsilon_0} \tag{27}$$

so that the imaginary component dominates ε. Under these circumstances it is possible to optimize the heat flow for all channels such that

$$\exp(-\alpha d) \approx 1. \tag{28}$$

The result that there is a maximum heat flow in a given channel links with more profound ideas of entropy flow. Some time ago [11] it was shown from very general arguments that the maximum flow of entropy in a single channel is linked to the flow of energy. Briefly the argument was that the flow of information in a channel is limited by

$$\dot{E} \geqslant \frac{3\hbar \ln^2 2}{\pi} \dot{I}^2 \tag{29}$$

where $\dot{E}$ is the energy flow and $\dot{I}$ the information flow. Identifying the energy flow with heat flow, $\dot{Q}$,

$$\dot{E} = \dot{Q} \qquad \dot{I} = \frac{\dot{Q}}{k_B T \ln 2} \tag{30}$$

we have

$$\dot{Q} \leqslant \frac{\pi k_B^2 T^2}{3\hbar} \tag{31}$$

hence as above,

$$[\dot{Q}_{EV}]_{\max} = \sum_k \frac{\pi k_B^2}{3\hbar} T^2 \tag{32}$$

assuming only p-polarized modes are active. Therefore our result is interpreted as conducting the maximum allowed amount of entropy per channel. Naturally if we sum over all channels there is a divergence unless there is a cut-off in k which would be given by the properties of the material.

5. Local heating of a surface by an STM tip

Let us suppose that a hot scanning tunnelling microscope tip travels over a cool surface, which may have molecules adsorbed upon it. If the heat flow is sufficiently great the surface may be raised to a substantial fraction of the tip's temperature, resulting in local desorption or decomposition of molecular species. Heat crosses from the tip to the surface via a tunnelling mechanism, just like the electrons responsible for the STM's operation, and similarly confined to the area of the surface immediately under the tip. Figure 5 illustrates the situation.

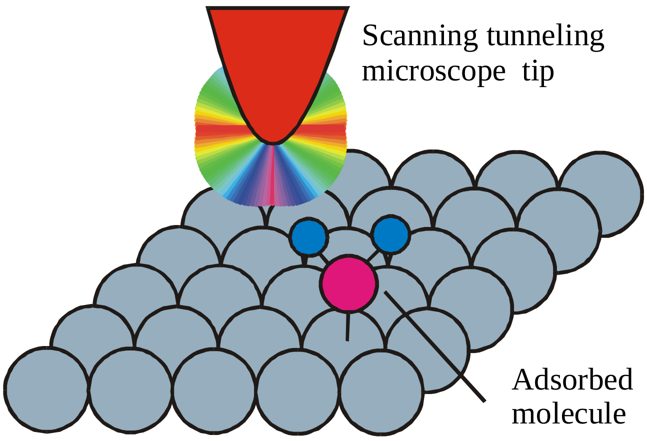

Figure 5. A hot scanning tunnelling microscope tip moves over a surface. Local heating may result in desorption of molecules. The STM tip is shown surrounded by the localized electromagnetic tunnelling modes.

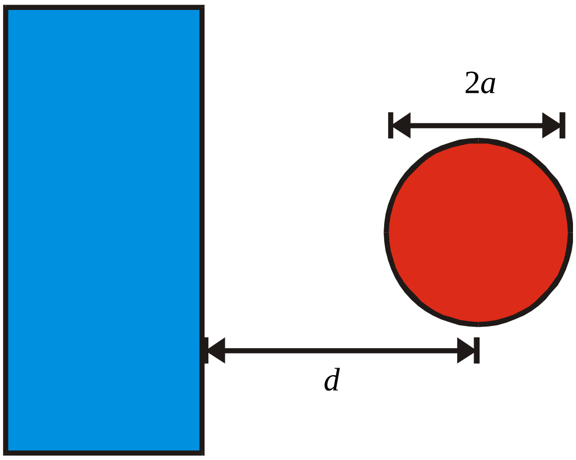

Figure 6. A small hot sphere heats a nearby surface via evanescent modes providing it is close enough.

The ability to introduce highly localized sources of heat, far more finely focused than could be achieved with a lens and free radiation, offers further possibilities for the STM to control local chemistry on a surface. Furthermore the STM offers the possibility of not only modifying the local surface conditions through heating, but also of observing the changes through subsequent scans.

To investigate the power of a hot tip to heat a surface we need some quantitative estimates of the heat flow. To this end we model the tip as a hot sphere of the same radius as the tip, see figure 6. This is a common approximation when calculating tunnelling current and the same arguments justify its use for calculating heat tunnelling. Therefore we wish to calculate the rate at which a hot sphere does work on a surface. For simplicity we assume that the sphere is positioned sufficiently far from the surface that only dipole modes of the sphere are excited. We also assume that we are close enough to the surface that we are in the electrostatic limit.

Hence,

$$a \ll d \tag{33}$$

and,

$$d \ll c_0 \omega^{-1}. \tag{34}$$

The contribution to the heat flow from tip to surface will be dominated by the tunnelling modes which in the appendix we calculate to be

$$\dot{Q}_{ST} \approx \frac{2\pi^3 a^3 k_B^4 T^4}{5 d^3 \hbar^3} \frac{\varepsilon_0^2}{\sigma_s \sigma} \tag{35}$$

where T is the tip temperature and we have assumed that

$$\sigma > \varepsilon_0 k_B T \hbar^{-1}. \tag{36}$$

If we assume the following parameters typical of a tungsten tip 1 nm above a tungsten surface,

$$\begin{aligned} T &= 300\ \mathrm{K} \\ d &= 10^{-9}\ \mathrm{m} \\ a &= 0.5\, d \\ \sigma_s &= \sigma = 10^5\ (\mathrm{m}\Omega)^{-1} \end{aligned} \tag{37}$$

and that the surface is cold so that there is no back flow of heat,

$$\dot{Q}_{ST} \approx 8.536 \times 10^{-17} \text{ watts} \tag{38}$$

the flow of heat will be confined to a very small area of diameter approximately the tip surface separation, d, and therefore the flux per unit area is

$$\dot{Q}_{ST}/\text{area} \approx \dot{Q}_{ST}/d^2 \approx 8.536 \text{ W m}^{-2}. \tag{39}$$

We could compare this to the heating when the surface is illuminated with black body radiation at 300 K, roughly taking account of the surface reflectivity by approximating

$$|R|^2 \approx \left|\frac{1-\sqrt{\varepsilon}}{1+\sqrt{\varepsilon}}\right|^2 = \left|\frac{1-\sqrt{1+\mathrm{i}\sigma_s/\omega\varepsilon_0}}{1+\sqrt{1+\mathrm{i}\sigma_s/\omega\varepsilon_0}}\right|^2 \approx 1-2\sqrt{\frac{2k_B T\varepsilon_0}{\hbar\sigma_s}}. \tag{40}$$

Hence the rate of heating from black body radiation is approximately

$$\dot{Q}_{BB}(1-|R|^2) \approx 4.5917 \times 10^2 \times 1.213 \times 10^{-2} \text{ W m}^{-2} = 5.57 \text{ W m}^{-2}. \tag{41}$$

Thus we expect to see some local enhancement of heating over that expected from uniform black body radiation.

However, and this is an important point, we can produce very much larger effects if the conductivity of the tip is tuned to the temperature. From (35) we see that the heating is inversely proportional to the conductivity of the tip provided that the inequality (36) for the conductivity is obeyed. In fact the effect will be a maximum when

$$\sigma \approx \varepsilon_0 k_B T \hbar^{-1} = 347.8 \ (\mathrm{m}\Omega)^{-1}. \tag{42}$$

Conductivities of this order are typical of semimetals such as carbon, or of composites. In fact only a thin coating of the right conductivity would be required and this might comprise an adsorbate from the gas phase. Assuming that we can achieve a tip with the requisite conductivity, substituting gives

$$\dot{Q}_{ST\max}/\text{area} \approx \frac{1.89 \times 10^7}{347.8} 85.36 \text{ W m}^{-2} = 4.64 \times 10^6 \text{ W m}^{-2}. \tag{43}$$

In other words by tuning the conductivity a tip can deliver a heat flow to the surface hugely enhanced relative to that available from black body radiation at the same temperature.

6. A nanoscale 'heat stamp'

It has recently been proposed [12] that near field optics could be exploited to write extremely fine details for integrated circuits. The concepts are similar to those outlined above: components of the electromagnetic field having short wavelength (and therefore the potential for high resolution) are naturally evanescent in nature and do not contribute to far field effects. Hence fine details in any patterned mask will rapidly dissolve away with distance from the mask. However if it is possible to position the wafer close to the mask then fine details can be resolved. Roughly speaking the separation between mask and wafer must be of the same order as the lateral detail to be resolved.

Once the propagating modes have been abandoned as the means of transporting radiation, the frequency plays little role in determining resolution which is now almost entirely dependent on separation. Therefore it is possible to imagine a mask consisting of a surface patterned alternately in highly reflecting (and therefore poorly emitting) material, and a second material chosen to optimize emission of heat into the evanescent modes as discussed above. A second surface placed under and very close to the mask will be preferentially heated beneath the active regions: see figure 7.

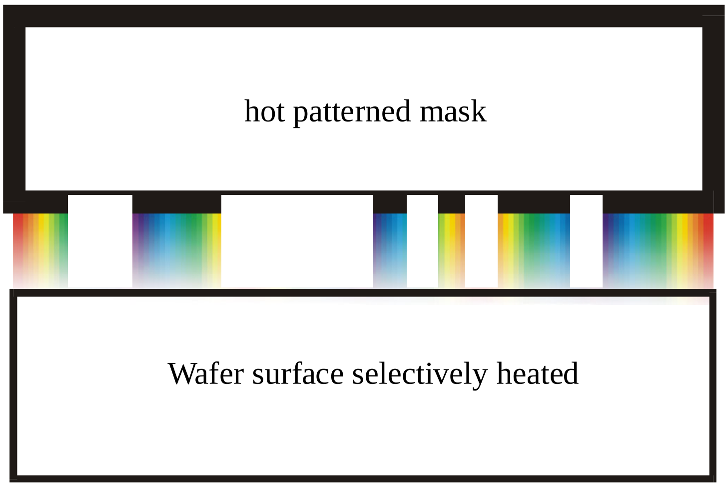

Figure 7. A mask is patterned using two materials: one that is a poor emitter of evanescent waves at the ambient temperature, another that is a good emitter. A wafer placed close to the mask will be selectively heated beneath the good emitter, an effect that may possibly be exploited to selectively etch the wafer to much higher resolution than possible with conventional methods.

This proposal leaves unanswered the question of how the two surfaces would be controlled to such a fine separation, but nevertheless offers additional possibilities for etching structures with very fine resolution.

7. Conclusions

The propagation of heat through a vacuum is strongly modified when surfaces come closer than the characteristic wavelength of the thermal radiation. At short distances new modes, the photon analogy of electron tunnelling in the STM, contribute to the heat current. For structures whose dimensions are just a few nanometres the tunnelling modes will usually dominate and give greatly enhanced heat flow particularly if the conductivity of the materials concerned is tuned to maximize the heat flow due to tunnelling. Characteristically at room temperature this implies conductivities typical of semimetals such as carbon, or of metal–insulator composites. In this way highly localized heating can be produced by an STM tip which may be exploited for the purpose of surface modification. An extension of this idea is suggested in the form of a ‘heat stamp’ that has nanometric resolution and may be exploited to etch wafers in much the same manner as the ‘optical stamp’ [12].

The concept of photon tunnelling between surfaces also links to more fundamental matters. Just as we can define the optimum emitter of free radiation as the perfect absorber, i.e. a black body, so we can define a surface designed to maximize the tunnelling of heat. The conditions are somewhat different but have a common origin in quantum information theory. The maximum heat flow through a single tunnelling mode is dictated only by the temperature and the fundamental constants.

Acknowledgment

I thank Gabriel Barton for pointing out several relevant references.

Appendix

We wish to calculate the rate at which a hot surface does work on a sphere positioned sufficiently far form the surface that only dipole modes are excited. We also assume that we are close enough to the surface that we are in the electrostatic limit. We can calculate the rate of cooling of a sphere using the same formula by reciprocity.

We require,

$$a \ll d \qquad d \ll c_0\omega^{-1}. \tag{A1}$$

First we calculate the heating of the sphere caused by an incident wave of the form

$$\boldsymbol{E} = E_{0p}\hat{\boldsymbol{K}}_p^+ \exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}_\parallel - \alpha z) \tag{A2}$$

where in the electrostatic limit,

$$\hat{\boldsymbol{K}}_p^+ = \frac{c_0}{\omega}[k_x k_y \mathrm{i}\alpha = +\mathrm{i}\sqrt{k_x^2 + k_y^2}]. \tag{A3}$$

Since we are in the electrostatic limit we can write the electric field in terms of a potential,

$$\phi = -E_{0p}|\hat{\boldsymbol{K}}_p^+| r\cos(\theta) \tag{A4}$$

where

$$|\hat{\boldsymbol{K}}_p^+|^2 = \hat{\boldsymbol{K}}_p^+\cdot\hat{\boldsymbol{K}}_p^{+*} = \frac{c_0^2(k^2+\alpha^2)}{\omega^2} \tag{A5}$$

and match fields inside,

$$E_{in}|\hat{\boldsymbol{K}}_p^+| r\cos(\theta) \tag{A6}$$

and outside,

$$E_{out}|\hat{\boldsymbol{K}}_p^+| r^{-2}\cos(\theta) - E_{0p}|\hat{\boldsymbol{K}}_p^+| r\cos(\theta) \tag{A7}$$

the sphere. Matching the field and its derivative on the surface of the sphere,

$$\begin{aligned} E_{in}a &= -E_{0p}a + E_{out}a^{-2} \\ \varepsilon E_{in} &= -E_{0p} - 2E_{out}a^{-3} \end{aligned} \tag{A8}$$

gives

$$\begin{aligned} E_{in} &= \frac{-3}{\varepsilon+2}E_{0p} \\ E_{out} &= \frac{-a^3}{3}(\varepsilon-1)E_{in} = \frac{+a^3}{3}\frac{\varepsilon-1}{\varepsilon+2}E_{0p}. \end{aligned} \tag{A9}$$

Rate of working:

$$\begin{aligned} P &= \int |\boldsymbol{E}|^2\omega\varepsilon_0 \,\mathrm{Im}\,\varepsilon\, \mathrm{d}^3\boldsymbol{r} = \omega\varepsilon_0\left|\frac{3}{\varepsilon+2}\right|^2 \mathrm{Im}\,\varepsilon\frac{4}{3}\pi a^3|E_{0p}|^2\frac{c_0^2(k^2+\alpha^2)}{\omega^2} \\ &= 12\pi a^3\omega\varepsilon_0\frac{\varepsilon_i}{(\varepsilon_r+2)^2+\varepsilon_i^2}|E_{0p}|^2\frac{c_0^2(k^2+\alpha^2)}{\omega^2}. \end{aligned} \tag{A10}$$

From earlier in the paper,

$$|E_{0p}|^2 = \hbar|\omega|\left[\frac{1}{2} + \frac{1}{\exp(\beta\hbar|\omega|)-1}\right]\frac{\omega\,\mathrm{Im}\,R_{1p}(k,\omega)}{\pi\varepsilon_0 c_0^2\alpha}\exp(-2\alpha d). \tag{A11}$$

Hence,

$$P = \frac{12a^3}{c_0^2\alpha}\frac{\varepsilon_i\,\mathrm{Im}\,R_{1p}}{(\varepsilon_r+2)^2+\varepsilon_i^2}\frac{c_0^2(k^2+\alpha^2)}{\omega^2}\frac{\hbar|\omega|\omega^2}{\exp(\beta\hbar|\omega|)-1}\exp(-2\alpha d). \tag{A12}$$

We now specialize to the case of a sphere of conductivity above a surface of conductivity σ_s so that

$$\varepsilon = 1 + \mathrm{i}\frac{\sigma}{\omega\varepsilon_0} \tag{A13}$$

$$\mathrm{Im}\, R_{1p}(k,\omega) = \mathrm{Im}\,\frac{\varepsilon_s - 1}{\varepsilon_s + 1} = \mathrm{Im}\,\frac{\mathrm{i}\sigma_s}{2\omega\varepsilon_0 + \mathrm{i}\sigma_s} \approx \frac{2\omega\varepsilon_0}{\sigma_s} \tag{A14}$$

where we have assumed that

$$\sigma \gg 2\omega\varepsilon_0 \qquad \sigma_s \gg 2\omega\varepsilon_0. \tag{A15}$$

Substituting,

$$\int_0^\infty P\,\mathrm{d}\omega \approx \frac{24a^3}{\alpha}\frac{\varepsilon_0^2}{\sigma_s\sigma}(k^2+\alpha^2)\exp(-2\alpha d)\hbar^{-3}k_B^4T^4\int_0^\infty \frac{x^3\,\mathrm{d}x}{\exp(x)-1}. \tag{A16}$$

Using

$$\int_0^\infty \frac{x^3\,\mathrm{d}x}{\exp(x)-1} = \frac{\pi^4}{15} \tag{A17}$$

we find

$$\int_0^\infty P\,\mathrm{d}\omega \approx \frac{8\pi^4a^3k_B^4T^4}{5\hbar^3}\frac{\varepsilon_0^2}{\sigma_s\sigma}(k^2+\alpha^2)\alpha^{-1}\exp(-2\alpha d). \tag{A18}$$

Finally,

$$\sum_{\boldsymbol{k}}\int_0^\infty P\,\mathrm{d}\omega \approx \frac{2\pi^3a^3k_B^4T^4}{5d^3\hbar^3}\frac{\varepsilon_0^2}{\sigma_s\sigma}. \tag{A19}$$

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